AD A033182

DIFFERENTIAL WEIGHTING IN MULTI-ATTRIBUTE UTILITY MEASUREMENT:

WHEN IT SHOULD NOT AND WHEN IT DOES MAKE A DIFFERENCE

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ADVANCED DECISION TECHNOLOGY PROGRAM

ARPA

CYBERNETICS TECHNOLOGY OFFICE
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TECHNICAL REPORT SSRI 76-6

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Sponsored by

Defense Advanced Research Projects Agency
ARPA Order No. 3052
Under Subcontract from
Decisions and Designs, Incorporated

August, 1976

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SUMMARY

Most important decisions involve choosing among alternatives with multiple value characteristics. A simple ten-step procedure has been proposed to help individuals and/or groups make practical decisions. This procedure is called multi-attribute utility analysis. One aspect of this procedure involves assigning importance weights to the attributes or dimensions of importance considered relevant to the decision. Some recent evidence has indicated that such differential weighting may not be necessary and that equal or unit weighting may be as good as far as making the final decision is concerned.

This paper explores some of the conditions under which differential weighting in multi-attribute utility analysis may or may not be appropriate. Two cases are considered: (1) For the case in which the attributes are not related or are related in a positive fashion (non-negatively correlated attributes), and under conditions when no well-defined criterion variable is available, differential weighting is not important. Unit or equal weighting will do just as well in the decision analysis. This means, for this case, multi-attribute utility analysis becomes even simpler since the weighting process need not be carried out. However, decision makers may wish to retain a form of weighting during the initial phase of the analysis since this sometimes helps in defining what attributes should be included in the analysis. In other words differential weighting may have psychological advantages even though nothing is to be gained numerically. (2) For the case of some or all of the attributes being negatively correlated, that is, more on one attribute means less on some other attribute, then differential weighting can make a difference. Thus, the final decision choice can be different when different weighting schemes are used.

An example of case 2 is given for the decision problem of choosing a "best" automobile from a set of automobiles. Some of the attributes considered important for making this decision might be such things as fuel economy, small exterior size, passing/acceleration ability, low interior noise, and so on. These attributes interact and tradeoffs are sometimes necessary. For example, in order to obtain excellent fuel economy, it might be necessary to sacrifice acceleration. This could be accomplished by considering lighter cars but this, in turn, could adversely affect ride quality, interior size, and so on. It was demonstrated that under these conditions three different weighting schemes led to different automobiles being considered as the "best."

The practical and theoretical implications of this result are discussed. For the case of negatively correlated attributes or a mixture of positive and negative correlations among the attributes, differential weighting makes a difference, and in practical situations this difference can be very important. This raises the intriguing question of just what weighting scheme should be used since this choice can critically affect the final outcome. Unfortunately, there is no theory to guide our thinking here. Research is continuing into developing such a theoretical rationale and empirical studies such as the one described in the report are also continuing.

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ACKNOWLEDGMENT

Support for this research performed by Social Science Research Institute was provided by the Advanced Research Projects Agency of the Department of Defense and was monitored under Contract N00014-76-C-0074 with the Office of Naval Research, under subcontract from Decisions and Designs, Inc.

DIFFERENTIAL WEIGHTING IN MULTI-ATTRIBUTE UTILITY MEASUREMENT: WHEN IT SHOULD NOT AND WHEN IT DOES MAKE A DIFFERENCE

INTRODUCTION

Most important decisions involve choosing among alternatives with multiple value characteristics. Consider, as an example, the decision problem of deciding which of several automobiles should be chosen as the "official car." Some of the attributes considered important in making this decision might be such things as fuel economy, small exterior size, large interior size, passing/acceleration ability, low interior noise, ride quality, and so on. These characteristics interact and trade-offs are often necessary. For example, in order to obtain excellent fuel economy, it might be necessary to sacrifice acceleration. This could be accomplished by considering lighter cars but this, in turn, could adversely affect ride quality, interior size, etc. Decision problems of this nature usually do not have any clear-cut criteria or objective solution on which to base the final decision. The choice of the decision alternative depends heavily on the preference structure of an individual or group of individuals. The entire process is based on human judgment.

A set of multi-attribute utility models (MAUM) have been proposed as an aid in making such decisions. Some of these are based on rather sophisticated mathematical techniques as represented by the works of Raiffa (1968) and Keeney and Raiffa (1976). These models are designed to "capture" the preference (utility) function of individual decision makers. This involves an assessment of the decision maker's preference values over the various attributes considered relevant to the decision and an assessment of the importance weights (scaling constants) for each of the attributes. Once this is accomplished, then there are several ways to aggregate these judgments to help select a reasonable if not best decision alternative. (See Keeney and Raiffa, 1976, for several interesting examples). Unfortunately, this assessment procedure, while formally elegant, is difficult and time consuming to implement in practice. For example, Keeney (1975), in an application to selection of energy policy alternatives where the attributes considered were such things as fatalities, pollution, land use, radioactive waste, etc., eleven attributes in all, reports that it took 8 hours to assess the utility function of a single person involved in the decision making process. In many decision making situations, it may be extremely difficult if not impossible to get high-level decision makers to take the time to go through this process. There are ways to speed up the process by using interactive computer programs (Keeney and Sicherman, 1975), but these can be expensive, require elaborate terminals, and still demand considerable time on the part of the decision makers.

For these and other reasons, Edwards and his associates (Edwards, 1971; Edwards and Guttentag, 1975; Edwards, Guttentag, and Snapper, 1975; Gardiner and Edwards, 1975) have proposed a simple ten-step procedure to help individuals or groups make practical decisions. This procedure lacks the mathematical sophistication of the Keeney-Raiffa models, but it does not require much time and effort and has been successfully applied to a wide variety of situations. Steps 5 and 6 involve direct numerical estimation of the importance weights to be assigned to the attributes and

Steps 7 and 8 involve direct judgments of the utilities or graphical assessment which in turn yields the numerical utilities.

The major portion of the remainder of this paper focuses on the problem of weighting in multi-attribute utility analysis. I will show first that for the case of all the attributes being correlated non-negatively, and such attributes are on standardized scales, then weighting is not important. This statement has an analytic and empirical foundation. If, on the other hand, some or all of the attributes are negatively correlated and such correlations cannot be removed by appropriate scaling, then differential weighting does make a difference. This will be demonstrated empirically since, to the best of my knowledge, no analytic solution exists for this case.

CASE 1

To set the stage for considering Case 1, non-negatively correlated attributes, consider first the general utility aggregation formula presented as 1:

$$U_j = \Sigma_{i=1} w_i u_i (a_{ij}) + f(u_1 (a_{1j}), \ldots, u_k (a_{kj})) \qquad (1)$$

where U_j is the overall utility for decision alternative j, $u_i(a_{ij})$ is the utility assigned to alternative j on attribute i, the w_i's are importance weights and the functional term f is a general term to be used in forming the composite U_j that allows for possible required interactions such as cross-product terms. In most practical situations, the f term is not needed, and Equation 1 becomes the simple additive model which Edwards recommends as Step 9 in his multi-attribute measurement scheme. Now for the case under consideration, any set of weights w_i are as good as any other; i.e., differential weighting is unimportant. The analytic proof of this is due to Wilks (1938), who demonstrated under reasonable conditions that the average value of the correlation between any two linear combinations of attributes differs from unity by terms on the order of 1/k where k is the number of attributes. The larger the value of positive correlations between pairs of attributes, the more rapidly the average value of the correlation between linear combinations approaches unity. This result is supported by more recent work by Dawes and Corrigan (1974), Einhorn and Hogarth (1975), Wainer (1976), and Wainer and Thissen (1976). This means that Step 6 of the Edwards procedure is irrelevant; one will do just as well by simply adding up the utilities (unit weighting). This also implies that the simple rank orderings of the attributes on a rough scale of importance (Step 5) is also irrelevant as the following analysis will demonstrate.

Analysis

Consider the case in which the decision-making system, a person or group of persons, rank orders the attributes on an importance scale and uses as the importance weight the actual rank so assigned. This means

for k attributes, one of them will get the rank (importance weight) of k, some other the rank of k-1 and so on until the last one gets a rank of 1. If any other positive numbers are used as the weights, for example, the relative importance weights obtained in Step 6 of the Edwards procedure, the result presented below will not be contradicted. I exclude the use of negative weights. Also, I assume the attributes can be placed on some standardized scale.

Now following Einhorn (1975), let

$$x_u = \sum_{i=1}^{k} x_i \tag{2}$$

$$x_w = \sum_{i=1}^{k} w_i x_i \tag{3}$$

where,

x_i = i^{th} attribute

w_i = weight for the i^{th} attribute ($i = 1, 2, \ldots, k$)

x_u = composite formed by unit weighting

x_w = composite formed by differential weighting

For convenience, we normalize the differential weights; i.e.,

$$\sum_{i=1}^{k} w_i = 1.0 \, .$$

This does not restrict the generality of the result to be presented below. Now, define $R^2_{x_u \cdot x_w}$ as the squared correlation between composites formed by unit and differential weighting measuring the similarity of the two composites. Ghiselli (1964) derives this quantity as

$$R^2_{x_u \cdot x_w} = \frac{(1 + (k-1)\bar{r}}{(1 + (k-1)\bar{r}) + \left(\dfrac{\sigma_w^2}{\bar{w}^2}\right)(1-\bar{r})} \tag{4}$$

where,

k = number of attributes

σ_w^2 = variance of weights

$\bar{w}^2$ = squared mean of weights

$\bar{r}$ = the average correlation between all pairs of attributes

It is assumed that all intercorrelations between attributes are non-negative.

The crucial part of (3) is the term $\frac{\sigma_w^2}{\bar{w}^2}$ in the denominator, which is the square of the coefficient of variation. Now since all the weights are ranks, we have

$$\bar{w}^2 = \left(\frac{1 + 2 + 3 + \ldots + k}{k}\right)^2$$

$$= \left(\frac{k(k + 1)}{2k}\right)^2 \qquad (5)$$

$$= \frac{(k + 1)^2}{4}$$

$$\sigma_w^2 = (1^2 + 2^2 + \ldots + k^2) - \bar{w}^2$$

$$= k\,\frac{(k+1)\ (2k+1)}{6} - \frac{(k+1)^2}{4}$$

$$= \frac{k^2 - 1}{12}. \qquad (6)$$

Equations (5) and (6) are the well-known expressions for the squared mean and variance of a set of ranks. If (5) and (6) are substituted for the values of $\sigma_{\bar{w}}^2$, and $\bar{w}^2$ in (4), then (4) can be evaluated as a function of $\bar{r}$ and k only. This I have done for $\bar{r}$ ranging from 0 to 1.0 and k ranging from 2 to 10.

The closer $\bar{r}$ is to 1.0 the higher $R^2_{x_u \cdot x_w}$ will be, and if $\bar{r} = 1.0$, the squared correlation between the unit and differential weighted composites will be 1.0 for all values of k. The squared correlation will decrease as $\bar{r}$ decreases and will be smallest when $\bar{r}$ equals 0. Thus, in Figure 1, $R^2_{x_u \cdot x_w}$ is presented only for the case $\bar{r} = 0$ as a function of k. Note that $R^2_{x_u \cdot x_w}$ never gets below about .79. In other words, the similarity between unit and differential weighting by ranks is quite high. In Figure 1, I have also plotted the minimum values for $R^2_{x_u \cdot x_w}$ as presented by Einhorn (1975). Using the result of a theorem proved by Katsnelson and Kotz (1957), Einhorn was presenting such minimum values for various values of $\bar{r}$ and k by showing that the coefficient of variation squared can never be greater than k-1. Thus, he substituted k-1 for σ_w^2 and numerically evaluated $R^2_{x_u \cdot x_w}$ for its minimum values. He notes that these would be <u>gross</u> under-estimations of $R^2_{x_u \cdot x_w}$ in actual situations. My result is closer to what can actually be expected in practice. It should be emphasized that the upper curve in Figure 1 with $\bar{r} = 0$ is for the case when the dissimilarity between the unit and differential weighting will be greatest.

CASE 2

Now consider the case in which some or all of the attributes are negatively correlated. There is no theory or analytic solution to guide us here. Newman, Seaver, and Edwards (1976), using a computer simulation technique and for the simplest possible case of just two attributes, however, demonstrate that differential weighting does make a difference when compared with simple unit weighting. The composite U_j of Equation 1 can be quite different for different combinations of weights and, therefore, affect the decision process considerably. As another illustration of this, I shall re-analyze a recently published result on how to select a "model car."

The Automobile Club's Target Car Program

The Automobile Club of Southern California has developed and is actively pursuing a Target Car program where the Target Car is defined as an optimum design goal for an automobile which would best meet a broad middle segment of the transportation needs of the motoring public. "The design should balance and optimize characteristics serving environmental, safety, and conservation goals" (McDonald, Bintz, and Banowet, 1975, p. 3).[1]

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[1] I am indebted to John W. McDonald and the Automobile Club of Southern California for permission to use their data.

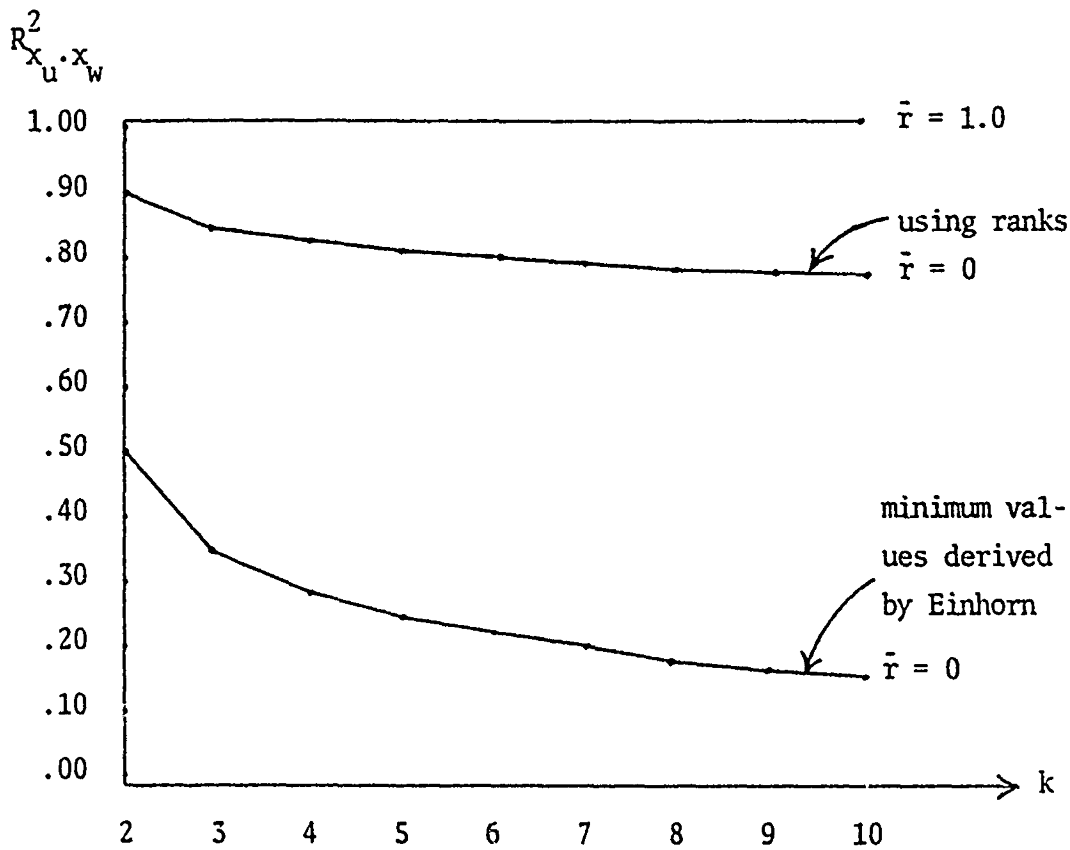


Figure 1. The Squared Correlation Between Unit and Differential Composites as a Function of the Average Inter-attribute Correlation ($\bar{r}$) and the Number of Attributes (k).

This particular example is chosen since the Auto Club used a procedure similar in spirit, if not in detail, to that recommended by Edwards. The attributes were identified by a group of automobile engineering experts and the driving public (Auto Club members). The attributes were rank ordered in importance, and importance weights were assigned to them. Each candidate automobile was given a value on each of the attributes. Finally, the composite value (utility) was calculated by a simple linear weighted sum. Since it is important for the re-analysis of the Auto Club data that will be done in this paper, I will describe their procedure in more detail.

After an extensive survey of the Club's membership and other expert opinion, eleven key target car characteristics or attributes were identified. These are listed below in order of importance:

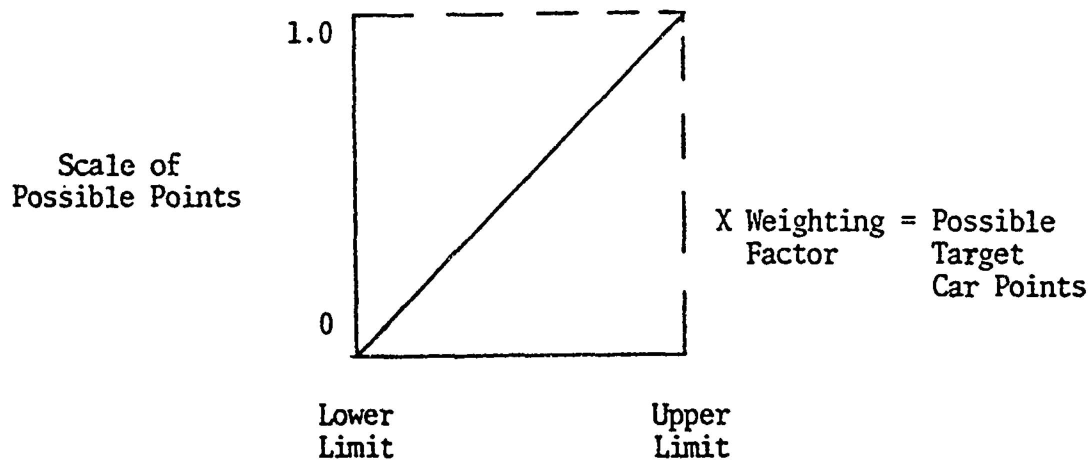
Fuel economy
Large interior size
Passing/acceleration ability
Low interior noise
Small exterior size
Crashworthiness
Luggage/parcel capacity
Handling
Ride quality
Ease of entry and exit
Maneuverability

The reader will note immediately that this list might not be exhaustive and perhaps several important characteristics were left out. The most obvious thing excluded is cost, a very important factor in vehicle design. Another excluded characteristic was styling. Cost was excluded, at least initially, since the Auto Club desired to seek an optimal design free from rigid cost constraints. Actually, cost was given separate consideration and will also be considered in this analysis. Styling was not considered since it was considered too subjective by the Auto Club. Actually whether the above list is considered appropriate or not for task selecting a vehicle is not germane to the topic under consideration since I am only concerned with the methodological issue of differential weighting.

For the objective characteristics in the above list, the Auto Club stipulated upper and lower values. If any candidate vehicle fell below the lower value in the measurement range, it was excluded from consideration. If, on the other hand, the vehicle fell at or above the upper limit, it was given the highest value. How the values were assigned and weighted is best explained by considering Figure 2.

Note in Figure 2 that the measurement ranges are oriented appropriately so that if high or low numbers are considered "good," then increasing or decreasing measurement can be given a value between 0 and 1. Technically, this means that the attributes are scaled to be monotonically increasing with total utility. Also, each characteristic is broken down into component

Figure 2: Car Points System



	Lower Limit		Upper Limit	Weighting Factor	Possible Target Car Points
Fuel Economy					
City	10	mpg	18	10	20
Highway	15	mpg	28	10	
Interior Size					
Front					
Leg	38	inches	42		
Head	35	inches	36	5	
Shoulder	37	inches	48		
Rear					15
Knee	24	inches	29		
Head	33	inches	36	10	
Shoulder	47	inches	48		
Performance					
Acceleration					
0-50 mph	15	seconds	10	4	9
40-60 mph	12	seconds	8	5	
Interior Noise					
30 mph	70	dbA	63	3	
55 mph	75	dbA	70	3	8
0-60 mph @ WOT	86	dbA	74	2	
Small Exterior Size	300	inches	240	7	7
Crashworthiness					
Front Crush	30	inches	60		
Sill Height	10	inches	16	6	6
Stroke	20	inches	30		
Door Thickness	5	inches	10		
Slalom Handling	8.2	seconds	6.2	5	5
Trunk Size	5	cubic feet	20	5	5
Ride Quality		subjective		5	5
Ease of Entry & Exit		subjective		5	5
Small Turning Circle	44	feet	35	3	3
					88

parts where appropriate and differentially weighted. The final weights are the composites given in the last column of Figure 2. Thus, Fuel Economy gets the highest weighting (20) and Small Turning Circle receives the lowest weight (3). Each candidate vehicle, if it met the minimum standards, was located on the horizontal scales of Figure 2 and the corresponding value read off the vertical graph.[2] Thus, for example, if a vehicle receives 14 mpg in the city and 22 mpg in the highway, that would locate it in the middle of the horizontal scale of Fuel Economy and thus receive a value of .50 on the vertical scale. This would be weighted (multiplied) by 20 and receive a total value of 10 on the attribute Fuel Economy. The total possible points that could be received by this procedure was 88 and this value defined the ideal target car for the Automobile Club. There were 24 vehicles that met or exceeded the minimum standards and received points under this system. The vehicle points, price, and price-per-point information are presented in Table 1. The vehicles, arranged in order of increasing price per point, are presented in Table 2. Table 1 enables a rating of the cars on the point system only. Table 2 presents an evaluation in terms of what it costs to obtain value points.

Before analyzing these data under different weighting schemes, we first obtained information on the intercorrelations of the attributes used to rate the vehicles. Since each of the 24 vehicles received a value on each of the 11 attributes plus the price data, it was possible to do this. The intercorrelation matrix is presented in Table 3.

The intercorrelations presented in Table 3 are about what one would expect in terms of positive and negative values. For example, the attributes of Passing/Acceleration, Low Interior Noise, and Crashworthiness all have moderately high negative correlations with Fuel Economy. Maneuverability is positively correlated with Fuel Economy and Small Exterior Size but negatively correlated with Crashworthiness. It should be remembered that all the numbered scales were oriented in the correct direction and this did not eliminate the negative correlations. I turn now to the basic question of this paper under such conditions: What effect does differential weighting have on the ordering of the test vehicles for selection decision purposes?

Different Weighting Schemes

Four different weighting schemes were investigated: (a) The original Auto Club point values (Auto Club); (b) The original Auto Club point values further weighted by the numerical rank assigned from high (11) to low (1) according to the rank ordering of the attributes in importance (thus, Fuel Economy was multiplied by 11, Large Interior Size was multiplied by 10 and so on down with Maneuverability receiving a weight of 1. This scheme amplified the original weights (Wgt. Auto Club)); (c) Unit weighting obtained by just adding up the values between 0 and 1 each vehicle received from Figure 2 on each of the 11 attributes (Unit); and finally, (d) each

[2]This procedure assumes all the utility functions were linear.

TABLE 1

Vehicle Points Showing Cost Data

Vehicle	Points	Price[a]	Price / Point
Volvo 164E	68	8,567	126
Mercedes 300D	67	13,078	195
Audi 100LS	64	7,058	110
Saab 99LE	61	6,847	112
Volkswagen Dasher	61	5,280	87
Toyoto Corona MK II	61	5,969	98
Mercedes 230	60	10,417	174
Audi Fox	59	5,678	96
BMW 530i	59	9,738	165
Datsun 610	58	4,766	82
Buick Century	55	5,558	101
Mazda RX4	54	5,207	96
Volkswagen Rabbit	54	4,353	81
AMC Matador	53	4,837	91
Toyota Corona	50	4,291	86
Ford Granada 6	49	4,992	102
Dodge Dart 6	49	5,209	106
AMC Pacer	48	4,567	95
Dodge Coronet 8	47	5,210	111
Ford Maverick 8	46	4,229	92
Chevrolet Nova LN 8	45	4,920	109
AMC Hornet 6	44	4,127	94
Ford Torino	44	5,380	122
Chrysler Cordoba	44	6,160	140

[a]Manufacturers' suggested retail price. Prices will vary depending on additional options chosen and discounts given.

TABLE 2

Test Vehicles in Order of Increasing $\frac{\text{Price}}{\text{Point}}$

Vehicle	$\frac{\text{Price}}{\text{Point}}$	Price[a]	Points
Volkswagen Rabbit	81	4,353	54
Datsun 610	82	4,766	58
Toyota Corona	86	4,291	50
Volkswagen Dasher	87	5,280	61
AMC Matador	91	4,837	53
Ford Maverick 8	92	4,229	46
AMC Hornet 6	94	4,127	44
AMC Pacer	95	4,569	48
Audi Fox	96	5,678	59
Mazda RX4	96	5,207	54
Toyota Corona MK II	98	5,969	61
Buick Century	101	5,558	56
Ford Granada 6	102	4,992	49
Dodge Dart 6	106	5,209	49
Chevrolet Nova LN 8	109	4,420	45
Audi 100LS	110	7,058	64
Dodge Coronet 8	111	5,210	47
Saab 99LE	112	6,847	61
Ford Torino	122	5,380	44
Volvo 164E	126	8,567	64
Chrysler Cordoba	140	6,160	44
BMW 530i	165	9,738	59
Mercedes 230	174	10,417	60
Mercedes 300D	195	13,078	67

[a]Manufacturers' suggested retail price. Prices will vary depending on additional options chosen and discounts given. Domestic car prices are typically discounted more than imported car prices.

TABLE 3

Intercorrelations of the Attributes

	1	2	3	4	5	6	7	8	9	10	11	12
1. FUEL ECONOMY												
2. LARGE INT. SIZE	.00											
3. PASSING/ACCELERATION	-.70	-.11										
4. LOW INTERIOR NOISE	-.65	.14	.31									
5. SMALL EXTERIOR SIZE	.73	-.23	-.42	-.64								
6. CRASHWORTHINESS	-.79	.09	.52	.66	-.90							
7. LUGGAGE CAPACITY	-.21	.53	.00	.12	-.40	.17						
8. HANDLING	.40	.46	-.34	-.28	.37	-.37	.24					
9. RIDE QUALITY	-.25	.32	-.12	.38	-.15	.20	.08	-.06				
10. EASE OF ENTRY AND EXIT	.24	.69	-.43	-.14	-.02	-.11	.56	.58	.14			
11. MANEUVERABILITY	.70	.07	-.43	-.42	.76	-.81	-.22	.35	.04	.23		
12. PRICE	.20	.70	-.31	.04	.05	-.14	.38	.40	.51	.64	.33	
13. PRICE/POINT	-.09	.57	-.17	.24	-.19	.13	.39	.15	.50	.48	.17	.92

value received from Figure 2 was weighted by the numerical rank assigned to the attributes in order of importance. Thus, Fuel Economy received a weight of 11, Large Interior Size a weight of 10, and so on down with Maneuverability receiving a rank of 1 (Wgt. Value). This was similar to (b) except the Auto Club weights were replaced by the numerical ranks.

There is no theoretical rationale for these weighting schemes. They are just different! However, if this were a practical decision-making problem, then the candidate vehicle would be rank ordered in terms of the composite value each would receive under the weighting scheme, and perhaps the top 2 or 3 candidates would be considered in the final decision.

Results

Rank order correlation coefficients (Kendall's Tau) were calculated among the four weighting schemes, and the results are presented in Table 4.

The coefficients presented in Table 4 are low enough to indicate that there are differences among the weighting schemes. These differences can affect the final decision as indicated by Tables 5 and 6. Table 5 orders the vehicles according to the composite value each vehicle receives under each weighting scheme disregarding price. Table 6 orders the vehicles according to the different composite values but under three different price categories. Just to make sure that different scaling procedures did not make any difference, the values in Table 6 are reported in standard score form with mean zero and unit standard deviation. For ease of reading, these standard scores had +3 added to them to eliminate negative values and were rounded to the first decimal place. In each table, the top three vehicles are indicated by the integers 1, 2, 3, respectively.

Tables 5 and 6 clearly indicate that differential weighting does make a difference for the case being considered, i.e, the existence of positive and negative correlations between the attributes considered important for the decision process. Although the selection of the top three vehicles is about the same for the Auto Club and Wgt. Auto Club; the Unit and Wgt. Value schemes result in different vehicles' being included in the top three. Thus, the final decision might well be different depending on what scheme was being used.

DISCUSSION

The results reported here are in excellent agreement with those who advocate simple unit weighting of relevant attributes in practical decision making situations for the case of non-negatively correlated attributes. This does not mean that in practical situations I would try to foist this idea onto a group of experts who were trying to make a difficult decision and were attaching importance weights to the attributes. After all, it is counter-intuitive to conceive of all attributes as being equally important even though, as has been demonstrated, this may well be

TABLE 4

Rank Order Correlation Between the Weighting Schemes

	1	2	3	4
1. Auto Club				
2. Wgt. Auto Club	.85			
3. Unit	.77	.59		
4. Wgt. Value	.70	.58	.75	

TABLE 5

Composite Values for the Vehicles Under Different Weighting Schemes[a]

Vehicle	Auto Club	Wgt. Auto Club	Unit[b]	Wgt. Value[b]
Volvo 164E	68 (1)	512 (1)	86 (2)	492 (1)
Mercedes 300D	67 (2)	505 (2)	83 (3)	457
Audi 100LS	64 (3)	483 (3)	81	448
Saab 99LE	61	453	78	423
Volkswagen Dasher	61	466	76	427
Toyota Corona MK II	61	452	81	470 (2)
Mercedes 230	60	425	81	423
Audi Fox	59	449	74	409
BMW 530i	59	390	87 (1)	460 (3)
Datsun 610	58	454	71	419
Buick Century	55	403	74	421
Mazda RX4	54	402	71	425
Volkswagen Rabbit	54	425	66	381
AMC Matador	53	388	69	423
Toyota Corona	50	415	57	356
Ford Granada 6	49	365	62	361
Dodge Dart 6	49	333	69	363
AMC Pacer	48	352	63	362
Dodge Coronet 8	47	344	62	386
Ford Maverick 8	46	326	64	369
Chevrolet Nova LN 8	45	340	59	368
AMC Hornet 6	44	330	57	335
Ford Torino	44	301	62	355
Chrysler Cordoba	44	327	57	369

[a]The values in parentheses indicate the top three vehicles

[b]These values were multiplied by 100 and rounded

TABLE 6

Composite Values for the Vehicles Under Different Weighting Schemes and Price Levels [a, b]

Price	Weighting Scheme			
Less than $5000	Auto Club	Wgt. Auto Club	Unit	Wgt. Value
Datsun 610	4.8 (1)	4.7 (1)	4.5 (1)	4.5 (2)
VW Rabbit	3.9 (2)	4.0 (2)	3.5 (3)	3.2 (3)
AMC Matador	3.7 (3)	3.2	4.2 (2)	4.6 (1)
Toyota Corona	3.1	3.8 (3)	1.7	2.3
Ford Granada 6	2.8	2.7	2.7	2.52
AMC Pacer	2.6	2.4	3.0	2.54
Ford Maverick 8	2.2	1.9	3.2	2.8
Chevrolet Nova 8	2.0	2.2	2.2	2.7
AMC Hornet	1.8	2.0	1.8	1.6
$5000-6000				
VW Dasher	4.1 (1.5)	4.2 (1)	3.8 (2)	3.5 (3)
Toyota Corona II	4.1 (1.5)	3.94 (2)	4.5 (1)	4.7 (1)
Audi Fox	3.8 (3)	3.89 (3)	3.42 (3)	3.0
Buick Century	3.2	3.15	3.4	3.3
Mazda RX4	3.0	3.13	2.98	3.5 (2)
Dodge Dart 6	2.3	2.0	2.7	1.8
Dodge Coronet	2.0	2.2	1.59	2.4
Ford Torino	1.5	1.5	1.63	1.6
Greater than $6000				
Volvo 164E	3.9 (1)	4.0 (1)	3.7 (2)	4.4 (1)
Mercedes 300D	3.8 (2)	3.9 (2)	3.4 (3)	3.46 (3)
Audi 100LS	3.4 (3)	3.6 (3)	3.16	3.23
Saab 99LE	3.1	3.2	2.9	2.60
Mercedes 230	2.9	2.7	3.2	2.58
BMW 530i	2.8	2.2	3.8 (1)	3.54 (2)
Chrysler Cordoba	.9	1.3	.8	1.2

[a]Numbers in the parentheses indicate the top three vehicles

[b]All numbers are standardized, had +3 added to them to eliminate negative values, and were rounded to one decimal place unless discrimination at the second decimal place was necessary

the case algebraically and numerically. Also there may be good psychological reasons for retaining ranking and assignment of importance weights in multi-attribute utility measurement. For example, this helps in providing a forum of debate about what attributes should be included or excluded. Perhaps in practical situations a post-sensitivity analysis of the decision-making process will indicate that unit weighting is as good as anything else. If this is done enough times, then the experts will get the message and devote more of their energies to defining what the relevant attributes should be and less time on differential weighting.

For the case of negatively correlated attributes or a mixture of positive and negative correlations then the story is different. Differential weighting does make a difference, and in practical situations this difference can be very important. This raises the intriguing question of just what weighting scheme should be used since the choice can critically affect the final outcome. Unfortunately, as mentioned previously, there is no theory to guide our thinking or suggest potentially useful research avenues that should be explored. Some recent work by Fishburn (1976) is directed toward filling this theoretical gap. Until this is further elaborated and understood, then research on the problem of differential weighting for the immediate future will have to be empirical, with each case being considered by itself.

REFERENCES

Dawes, R.M. and Corrigan, B. Linear models in decision making. Psychological Bulletin, 1974, 81, 95-106.

Edwards, W. Social Utilities. In Decision and Risk Analysis: Powerful New Tools for Management. Proceedings of the Sixth Triennial Symposium, June 1971, Hoboken: The Engineering Economist, 1972, 119-129.

Edwards, W. and Guttentag, M. Experiments and Evaluations: A Re-Examination. Chapter in Bennett, C. and Lumsdaine, A. (Eds.), Experiments and Evaluations. New York: Academic Press, 1975.

Edwards, W., Guttentag, M., and Snapper, K. Effective Evaluation: A Decision Theoretic Approach. In Streuning, E.L., and Guttentag, M. (Eds.), Handbook of Evaluation Research, Vol. 1, Beverly Hills, Ca.: Sage Publications, 1975.

Einhorn, H.J. and Hogarth, R.M. Unit weighting schemes for decision making. Organizational Behavior and Human Performance, 1975, 13, 171-192.

Einhorn, H. Equal weighting in multi-attribute models: A rationale, an example, and some extensions. Unpublished paper, School of Business, University of Chicago, 1975.

Fishburn, P.D. Approximations of two-attribute utility functions. Mimeographed manuscript, College of Business Administration, The Pennsylvania State University, 1976.

Gardiner, P. and Edwards, W. Public Values: Multi-Attribute Utility Measurement for Social Decision Making. In Kaplan, J., and Schwartz, S. (Eds.), Human Judgment and Decision Processes, New York: Academic Press, 1975.

Ghiselli, E.E. Theory of Psychological Measurement. New York: McGraw-Hill, 1964.

Katsnelson, J. and Kotz, S. On the upper limits of some measures of variability. Archiv fur Meteorologie, Geophysic and Bioclimatologie, 1957, 8, 101-107.

Keeney, R.L. Energy policy and value tradeoffs. International Institute for Applied Systems Analysis, Research Memorandum, December 1975.

Keeney, R.L. and Sicherman, A. An interactive computer program for assessing and analyzing preferences concerning multiple objectives. International Institute for Applied Systems Analysis, Research Memorandum, 1975.

Keeney, R.L. and Raiffa, H. Decisions with Multiple Objectives. New York: Wiley, to be published (1976).

McDonald, J.W., Bintz, L.J., and Banowet, R.A. The Target Car Program. Automobile Club of Southern California, 1976.

Newman, J.R., Seaver, D., and Edwards, W. Unit versus differential weighting schemes for decision making: A method of study and some preliminary results. University of Southern California, Social Science Research Institute Research Report, SSRI 76-6, 1976.

Raiffa, H. Decision Analysis: Introductory Lectures on Choices Under Uncertainty. Reading, Mass.: Addison-Wesley, 1968.

Wainer, H. Estimating coefficients in linear models: It don't make no nevermind. Psychological Bulletin, 1976, 83, 213-217.

Wainer, H. and Thissen, D. Three steps towards robust regression. Psychometrika, 1976, 41, 9-33.

Wilks, S.S. Weighting systems for linear functions of correlated variables when there is no dependent variable. Psychometrika, 1938, 3, 23-40.

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Unclassified

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE

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1. REPORT NUMBER
[redacted]

2. GOVT ACCESSION NO.

3. RECIPIENT'S CATALOG NUMBER

9

4. TITLE (and Subtitle)

6

Differential Weighting in Multi-Attribute Utility Measurement: When it Should Not and When it Does Make a Difference,

5. TYPE OF REPORT & PERIOD COVERED

Technical rept.

6. PERFORMING ORG. REPORT NUMBER

SSRI 76-6

7. AUTHOR(s)

10 J. Robert Newman

8. CONTRACT OR GRANT NUMBER(s)

N00014-76-C0074

Sub. 75-030-0711

9. PERFORMING ORGANIZATION NAME AND ADDRESS

Social Science Research Institute
University of Southern California
Los Angeles, California 90007

10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS

ARPA Order No. 3052

11. CONTROLLING OFFICE NAME AND ADDRESS

Advanced Research Projects Agency
1400 Wilson Boulevard
Arlington, Virginia 22209

12. REPORT DATE

11 Aug[redacted]t [redacted]976

13. NUMBER OF PAGES

30

14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)

Office of Naval Research
800 North Quincy Street
Arlington, VA 22217

12 29p.

15. SECURITY CLASS. (of this report)

Unclassified

15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for Public Release; Distribution Unlimited

15 N00014-76-C-0074,

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

ARPA Order-3052

18. SUPPLEMENTARY NOTES

Support for this research performed by Social Science Research Institute was provided by the Advanced Research Projects Agency of the Department of Defense and was monitored under Contract N00014-76-C-0074 with the Office of Naval Research, under subcontract from Decisions and Designs, Inc.

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Decision Analysis
Multi-attribute utility analysis
Differential weighting schemes

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Some of the conditions under which differential weighting in multi-attribute utility measurement may be used are explored. For the case of non-negatively correlated attributes, and under conditions when no well-defined criterion variable is available, differential weighting is not important. Unit or equal weighting will do just as well. For the case of some or all of the attributes being negatively correlated and when the negative signs cannot be

(cont'd p 24)

DD FORM 1473, 1 JAN 73 EDITION OF 1 NOV 65 IS OBSOLETE
S/N 0102-LF 014-6601

Unclassified

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

4/B

390664

(Cont.)

removed by appropriate scaling, then differential weighting can make a difference. Thus, the final choice can be quite different depending upon what weighting scheme is used. An example of choosing a "best" automobile from a set of automobiles is given. The practical implications of these results are discussed. Decision makers may wish to retain differential weighting even for the case in which it does not make a difference, but for psychological, not numerical reasons.